Optimal Control of the Magneto-hydrodynamic Generator at the Scramjet Inlet

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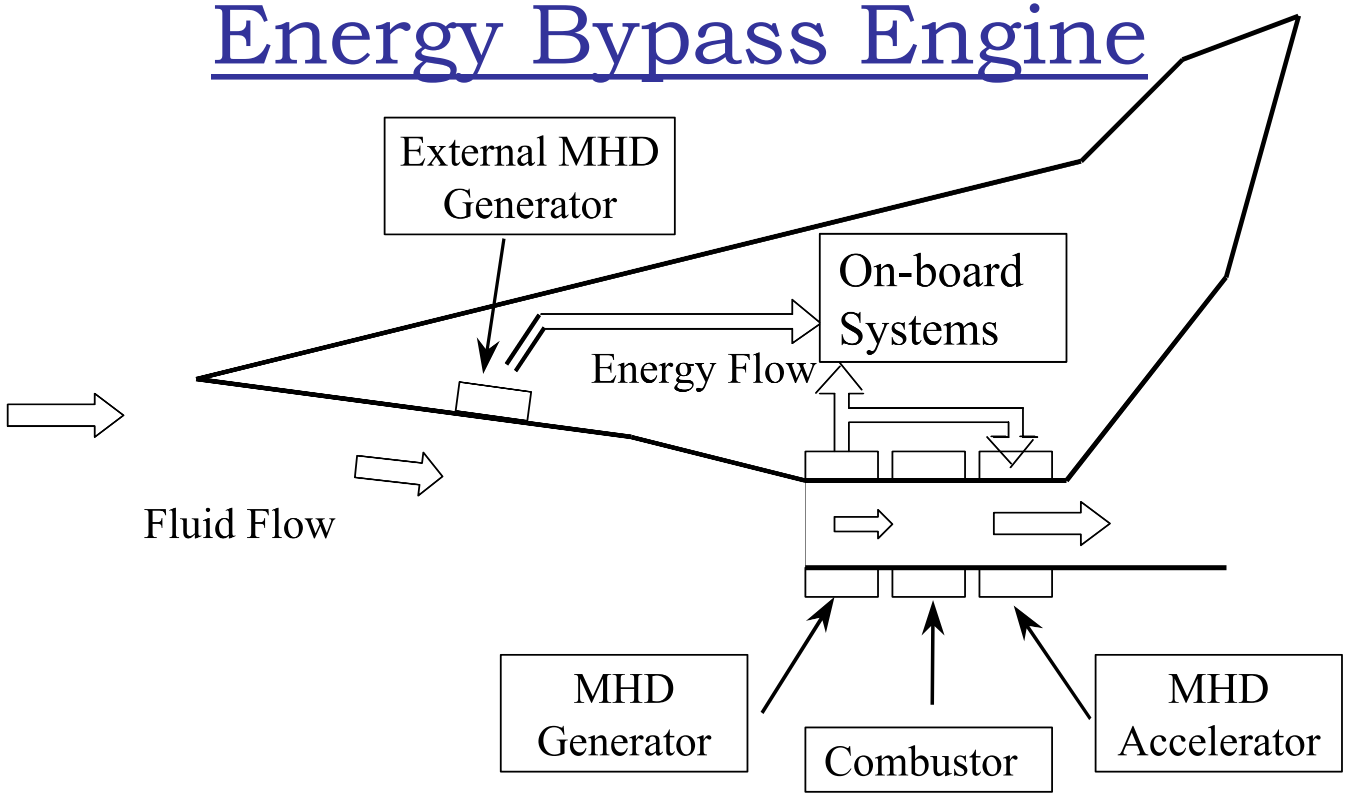
Presentation Outline

- The AJAX concept
- Analytical modeling
- The role of control
- Cost-to-go design for optimal control using Neural Networks
- Implementation details
- Results
- State Feedback Control Architecture
- Conclusions

The Magneto-hydrodynamic (MHD) Energy Bypass Engine



Schematic of some of the technologies envisioned in the AJAX

1) Fraishtadt, V.L., Kuranov, A.L., and Sheikin, E.G., "Use of MHD Systems in Hypersonic Aircraft," Technical Physics, Vol. 43, No.11, 1998, p.1309.
2) Gurijanov, E.P., and Harsha, P. T., "AJAX: New Directions in Hypersonic Technology," AIAA Paper 96-4609, 1996.

Analytical Model

- Assumptions:
  - One-dimensional steady state flow
  - Inviscid flow
  - No reactive chemistry
  - Low Magnetic Reynolds number
- '*x-t*' equivalence

Flow Equations

- Continuity Equation:

$$\frac{d(\rho u A)}{dx} = 0$$

x - coordinate along the channel

ρ - Fluid density

u - Fluid velocity

A - Channel cross-section area

- Force Equation:

$$\rho u \frac{du}{dx} + \frac{dP}{dx} = -(1-k)\sigma u B^2$$

P - Fluid pressure

k - Load factor

σ - Fluid conductivity

B - Magnetic field

Flow Equations…

- Energy Equation:

$$\rho u \frac{d(\gamma\varepsilon + \frac{u^2}{2})}{dx} = -k(1-k)\sigma u^2 B^2 + Q_\beta$$

ε- - Fluid internal energy

Q_β - Energy deposited by the e-beam

- Continuity Equation for the electron number density:

$$\frac{d(n_e u)}{dx} = \frac{2 j_b \varepsilon_b}{e Y_i Z} - \beta n_e^2$$

n_e - Electron number density

j_b - Electron beam current

ε_b - E-beam energy

Z- Channel width

The Role of Control

- Electron beam current as the control element
- Maximizing energy extraction while minimizing energy spent on the e-beam ionization
- Minimizing adverse pressure gradients
- Attaining prescribed values of flow variables at the channel exit
- Minimizing the entropy rise in the channel

Performance Index

- Minimize

$$J=\begin{bmatrix} T(x_f)-T_e & M(x_f)-M_e \end{bmatrix}\begin{bmatrix} p_{11} & 0 \\ 0 & p_{22} \end{bmatrix}\begin{bmatrix} T(x_f)-T_e \\ M(x_f)-M \end{bmatrix}+p_{33}\left\{C_v \ln\left[\frac{P(x_f)}{\rho(x_f)^{\gamma}}\right]-C_v \ln\left[\frac{P(0)}{\rho(0)^{\gamma}}\right]\right\}$$

$$+\int_0^{x_f}\left\{\left[\frac{q_{11}Q_{\beta}A-q_{22}k(1-k)\sigma u^2 B^2 A}{\rho u A}\right]+q_{33}(x)g(P)\right\}dx$$

Motivating the Cost-to-go Approach

- We need a control approach that can
  - work for nonlinear systems
  - be data-based
  - be applicable to infinite as well as finite horizon problems
  - easily adaptable

Motivating the Cost-to-go Approach…

Linear time invariant system:

$$x(k+1) = Ax(k) + Bu(k)$$

Parameterizing,

$$u(k) = Gx(k)$$

$$\Downarrow$$

$$x(k+i) = (A+BG)^i x(k)$$

$$\Downarrow$$

$$V(k,G) = \frac{1}{2}\sum_{i=1}^{r}[x(k+i)^T Q x(k+i) + u(k+i-1)^T R u(k+i-1)]$$

$$= \frac{1}{2} x(k)^T \sum_{i=1}^{r}[(A+BG)^{i^T} Q (A+BG)^i + (A+BG)^{i-1^T} G^T R G (A+BG)^{i-1}] x(k)$$

Modified Approach:

Parameterize as,

$$u(k) = G_1 x(k)$$

$$u(k+1) = G_2 x(k)$$

$$\ldots$$

$$u(k+r-1) = G_r x(k)$$

$$\Downarrow$$

$$x(k+i) = (A^i + A^{i-1}BG_1 + \ldots + ABG_{i-1} + BG_i)x(k)$$

$$\Downarrow$$

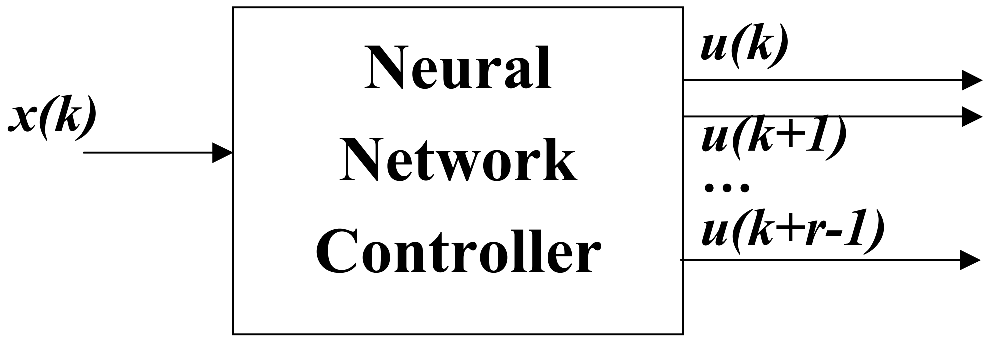
$$V(k) = \frac{1}{2}x(k)^T[(A^r + \ldots + ABG_{r-1} + BG_r)^T Q(A^r + \ldots + ABG_{r-1} + BG_r)$$

$$+ \ldots + (A + BG_1)^T Q(A + BG_1) + G_r{}^T RG_r + \ldots + G_1{}^T RG_1]x(k)$$

Solution with a unique minimum

Formulation of the Control Architecture: NN Controller

- Use of the modified approach to formulate the control architecture
- Instead of a single controller structure *(G)*, need *'r'* controller structures
- Outputs of the *'r'* controller structures, generate *u(k)* through *u(k+r-1)*
- Parameterize the *'r'* controller structures using an effective Neural Network



Formulation of the Control Architecture: NN Cost-to-go Function Approximator

- Parameterize the cost-to-go function using a Neural Network (*CGA* Neural Network)
- Inputs to the *CGA* Network:

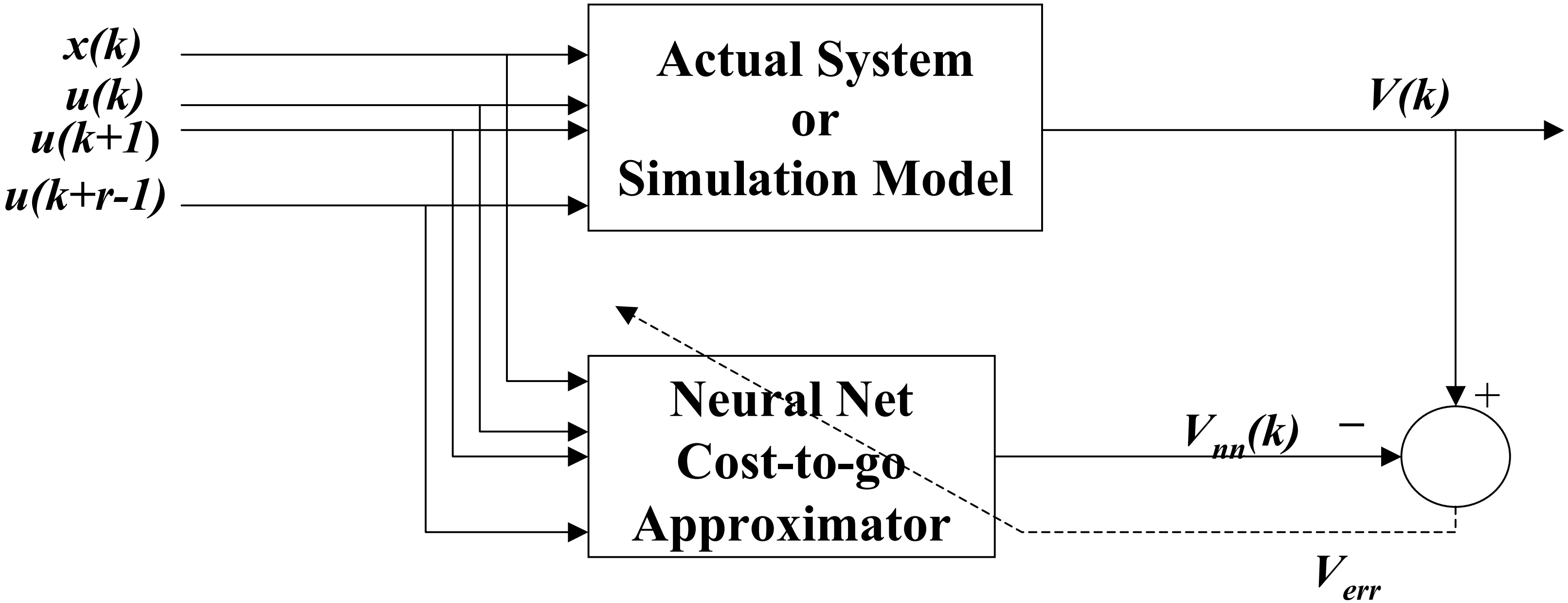
$$x(k),\ u(k), \ldots, u(k+r-1)$$

- Use the analytical model, or a computer simulation or the physical model to generate the future states.
- Use the *'r'* control values and the *'r'* future states to get the ideal cost-to-go function estimate.

$$V(k) = \frac{1}{2}\sum_{i=1}^{r}[x(k+i)^T Qx(k+i) + u(k+i-1)^T Ru(k+i-1)]$$

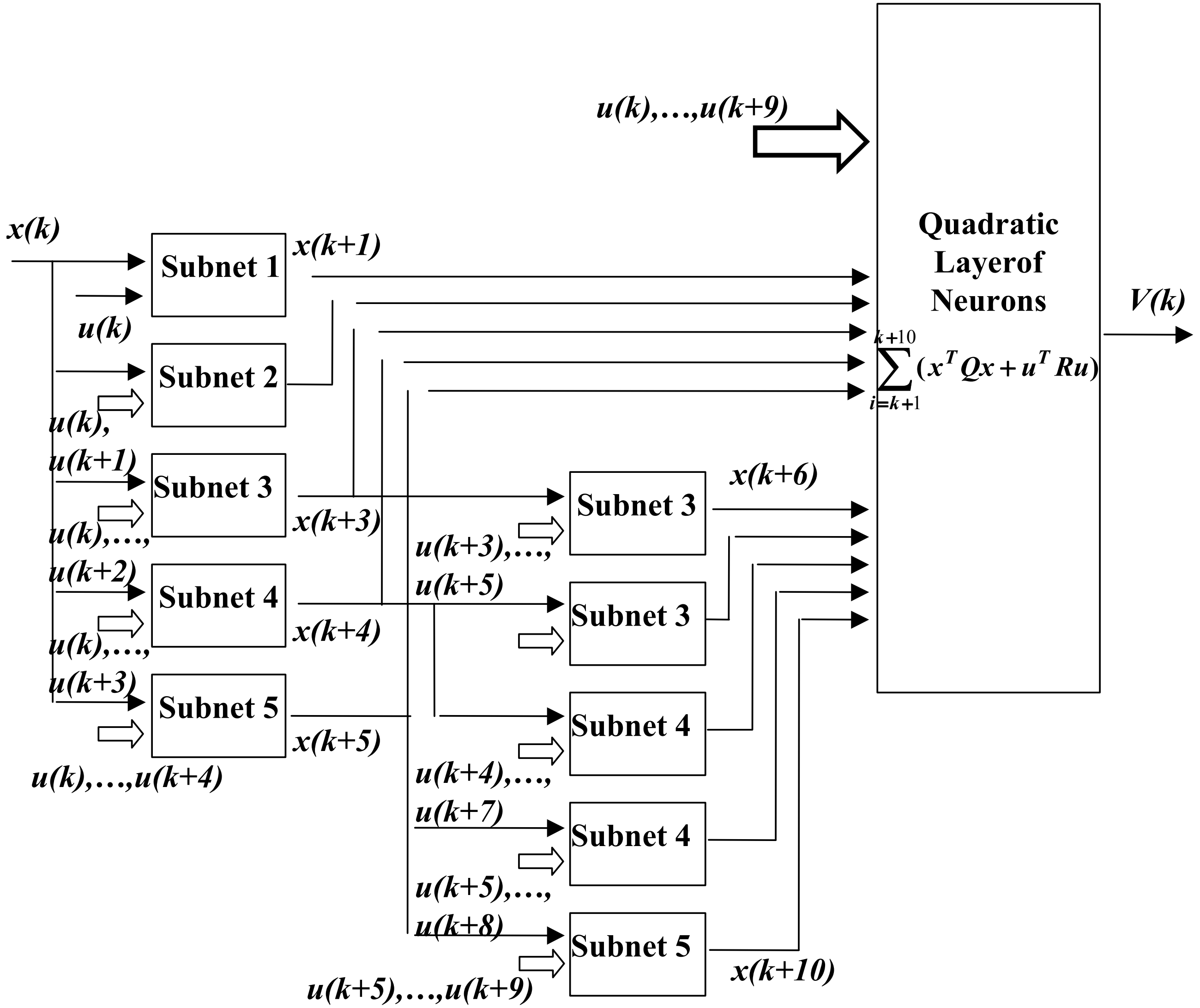
- Use this to train the *CGA* Neural Network

CGA Neural Network Training



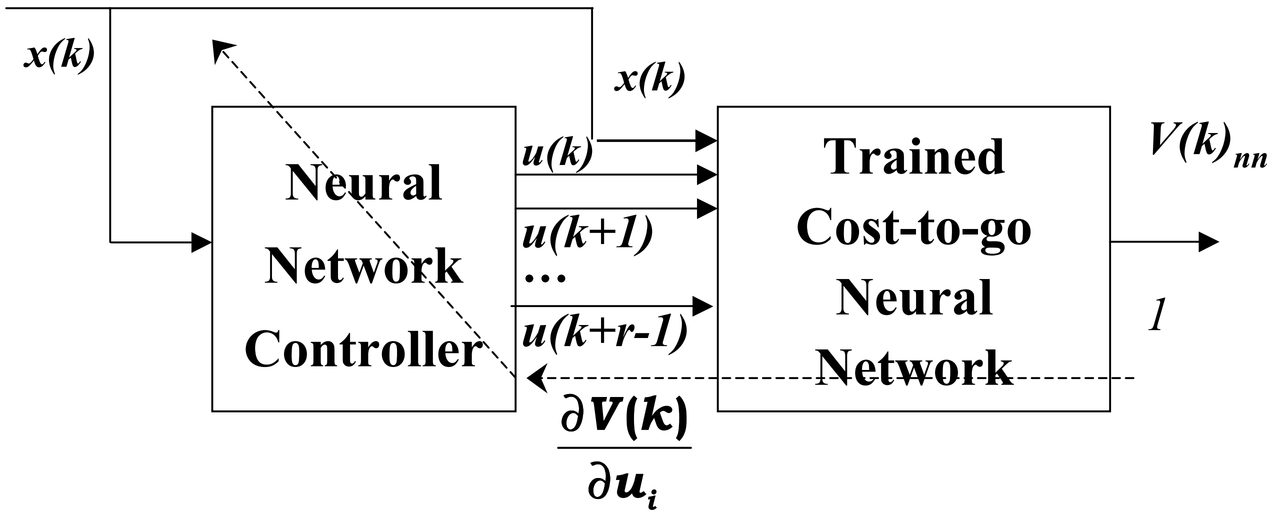
Neural Network Cost-to-go Approximator Training

Bringing Structure to the CGA Network



Implementation of the Hybrid CGA Network of order '*r* = *10*', using trained subnets of order *1* through *5*

Neural Network Controller Training



- Gradient of $V(k)$ with respect to the control inputs $u(k)$,…, $u(k + r - 1)$ is calculated using back-propagation through the *'CGA'* Neural Network.
- These gradients can be further back-propagated through the Neural Network controller to get, $\frac{\partial V(k)}{\partial G_1}$ through $\frac{\partial V(k)}{\partial G_r}$
- Neural Network controller is trained so that

$$\frac{\partial V(k)}{\partial G_i} \rightarrow 0, i = 1 \ldots r$$

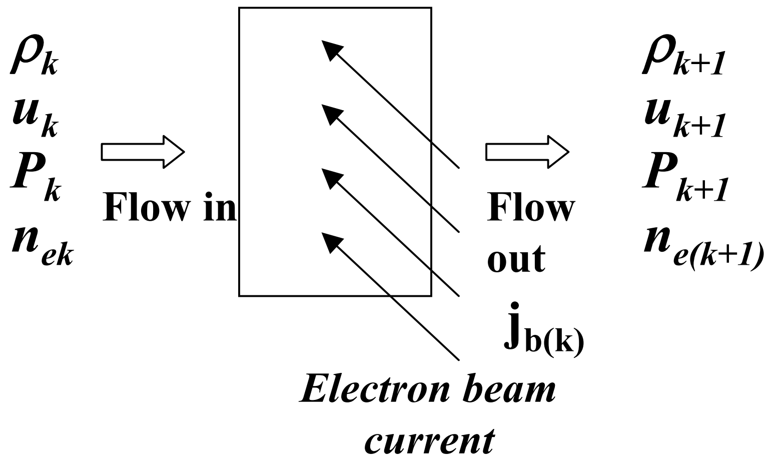
Salient Features of the Formulation

- Simplification of the optimization problem
- Decoupled CGA network training and the controller network training
- Introduction of structure in the CGA network
- Same basic architecture for linear or nonlinear systems.
- Data-based implementation - No explicit analytical model needed
- Adaptive control architecture with the use of Neural Networks

Translating the Approach to the MHD problem

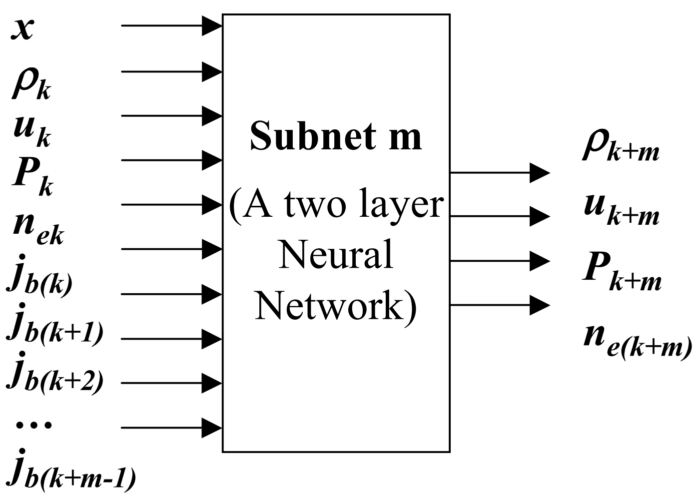
- In terms of the *'x-t'* equivalence, the problem is time-dependent
- Optimization equivalent to the fixed end time optimal control
- Procedure:
  - Defining subnets
  - Parameterizing and training the subnets
  - Arranging them together to get the cost-to-go function *V(0)*
  - Parameterizing and training the Neural Network controller

Defining and Parameterizing the Subnets



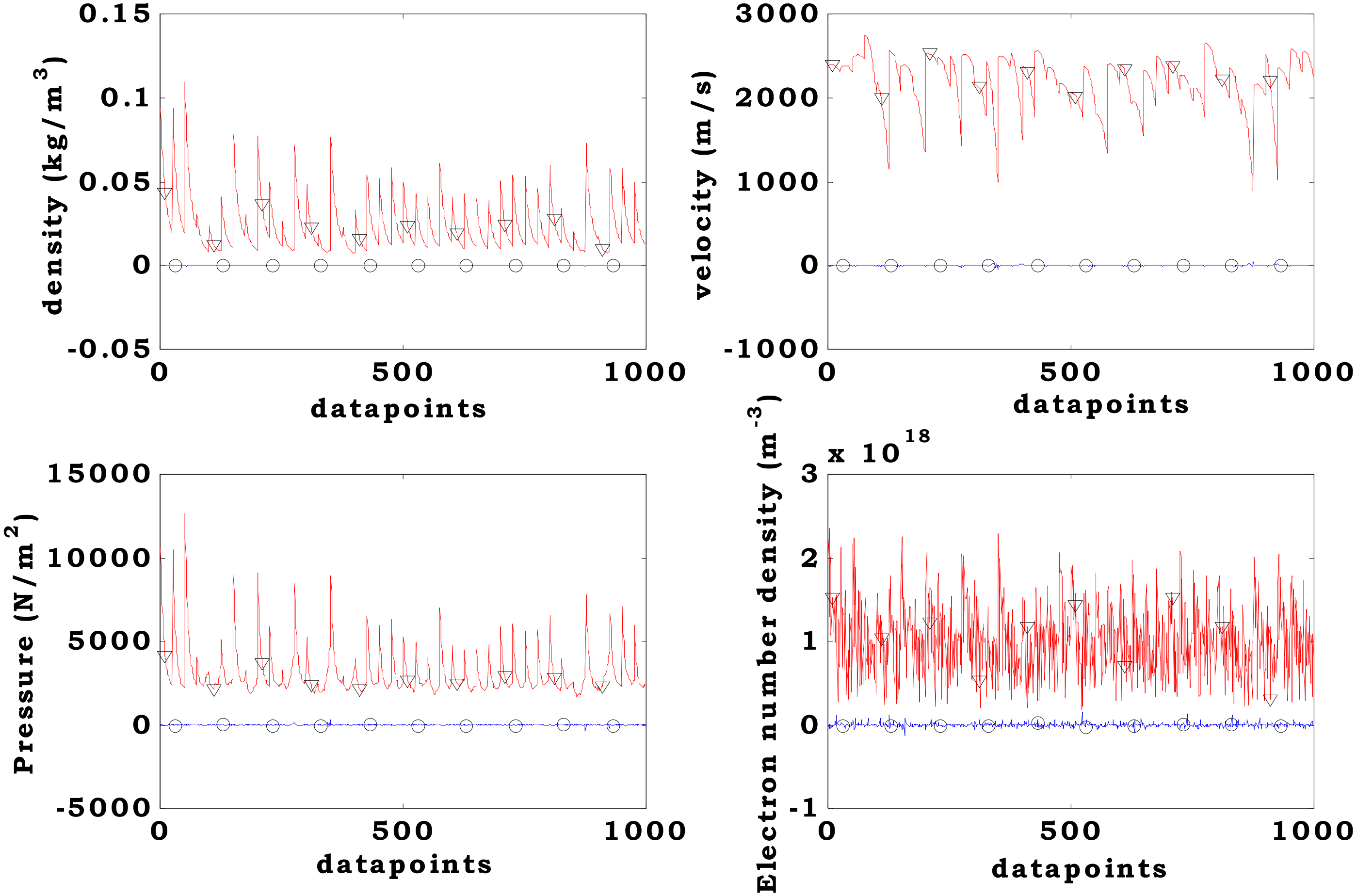
Physical picture describing Subnet 1

- Continuously spaced e-beam windows each having a length of 1 cm
- Subnet 1 chosen to correspond to the system dynamics between a group of 4 e-beam windows
- Length of the channel = 1 m
- Need subnets upto order 25



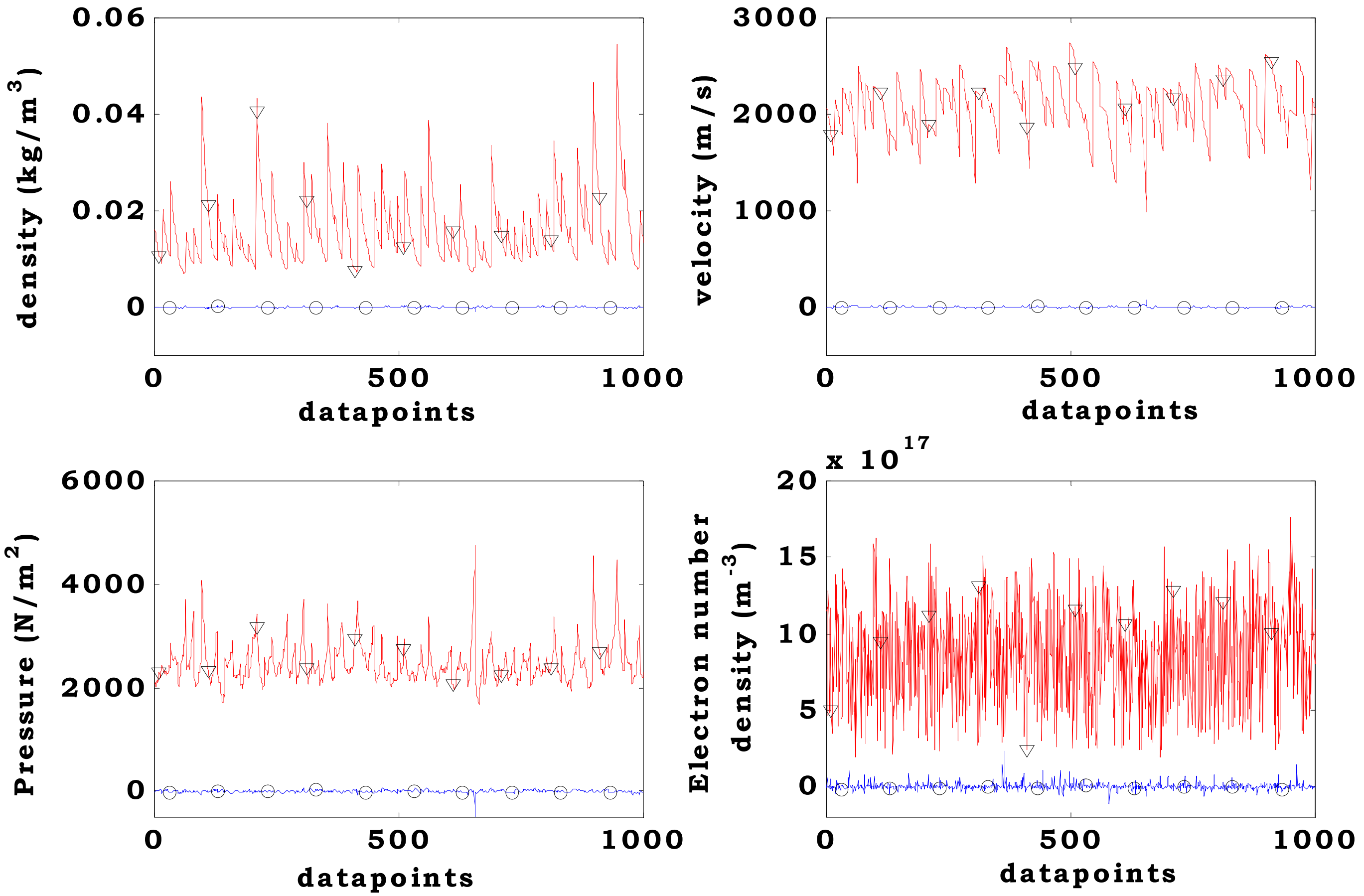
Subnet *m*, inputs and outputs.

Training Results for Subnet 1



Testing Subnet 1, '∇'- Ouput value given by subnet 1, 'o' – Error between the subnet 1 output and the ideal value given by the simulation

Training Results for Subnet 10



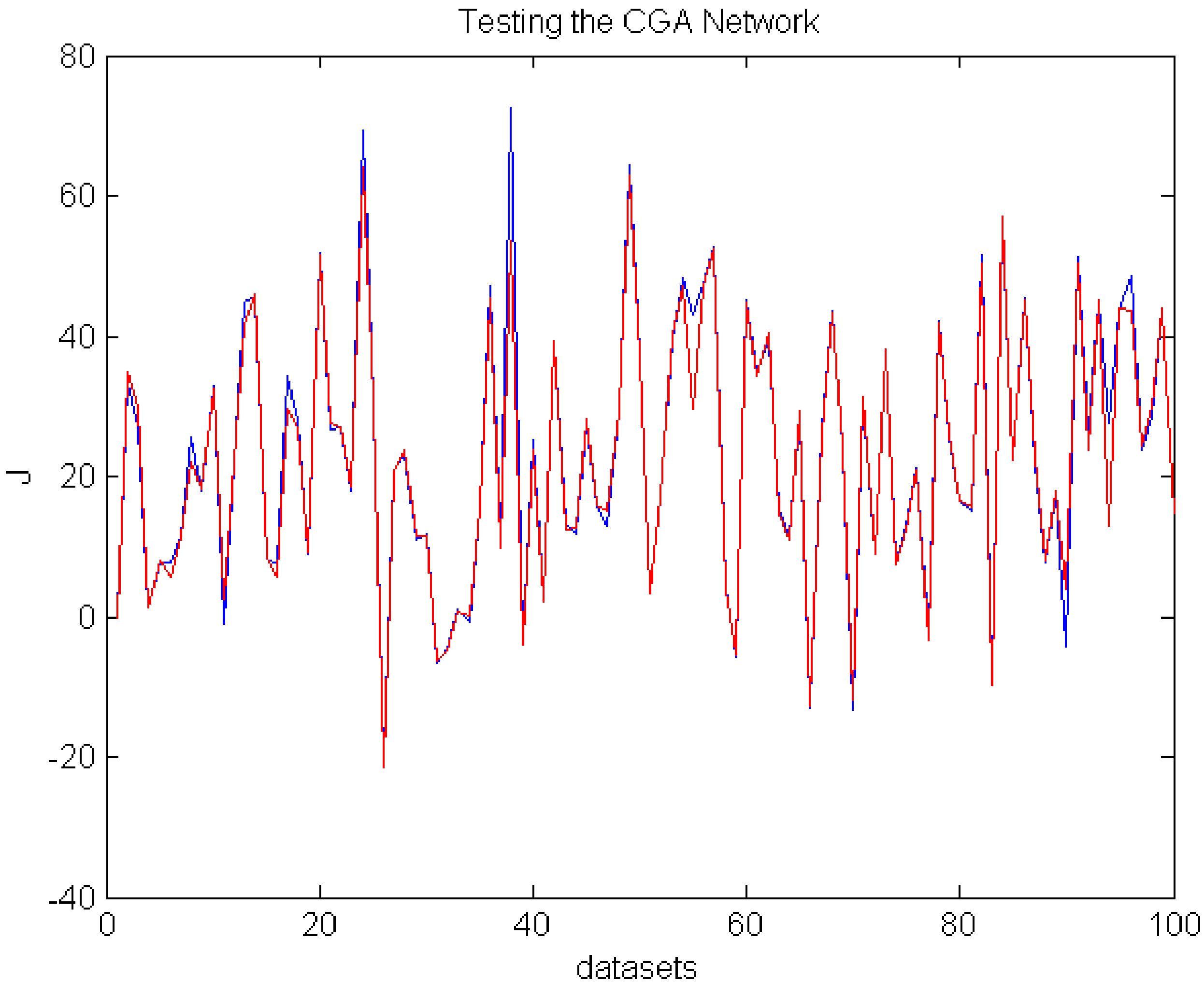
Testing Subnet 10, '∇'- Ouput value given by subnet 10, 'o' – Error between the subnet 10 output and the ideal value given by the simulation

Building the Cost-to-go Approximator

- Choice of the weighting coefficients (q11,q22,q33,p11,p22,p33)
  - Energy extracted-Energy used
  - Pressure Profile
  - Terminal conditions
- Translation of the integral into a summation.

Testing the CGA Network

q11 =1e-3, q22 = 1e-4, q33 = 0, p11 = 1e-4, p22 = 1, p33 = 1e-2

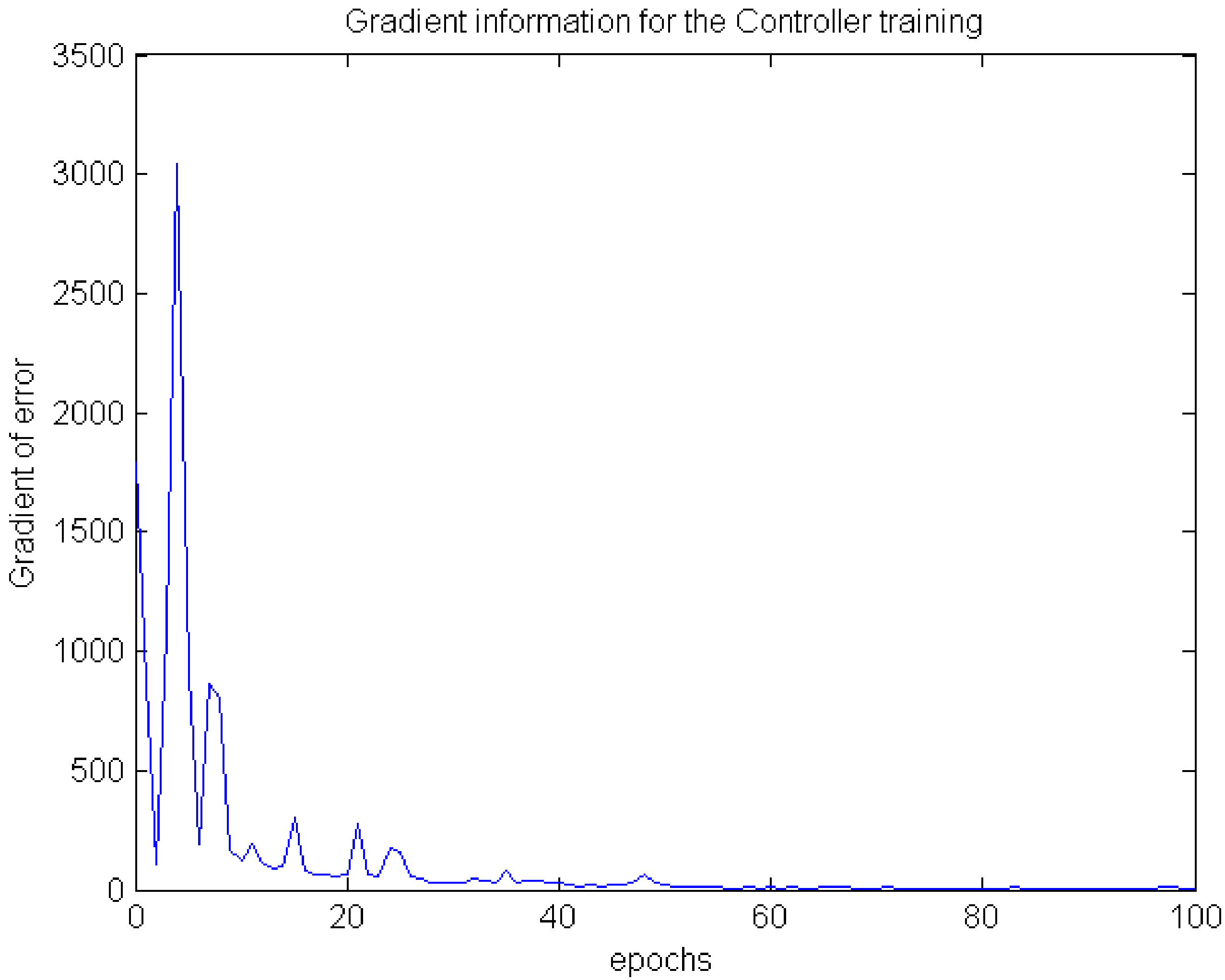


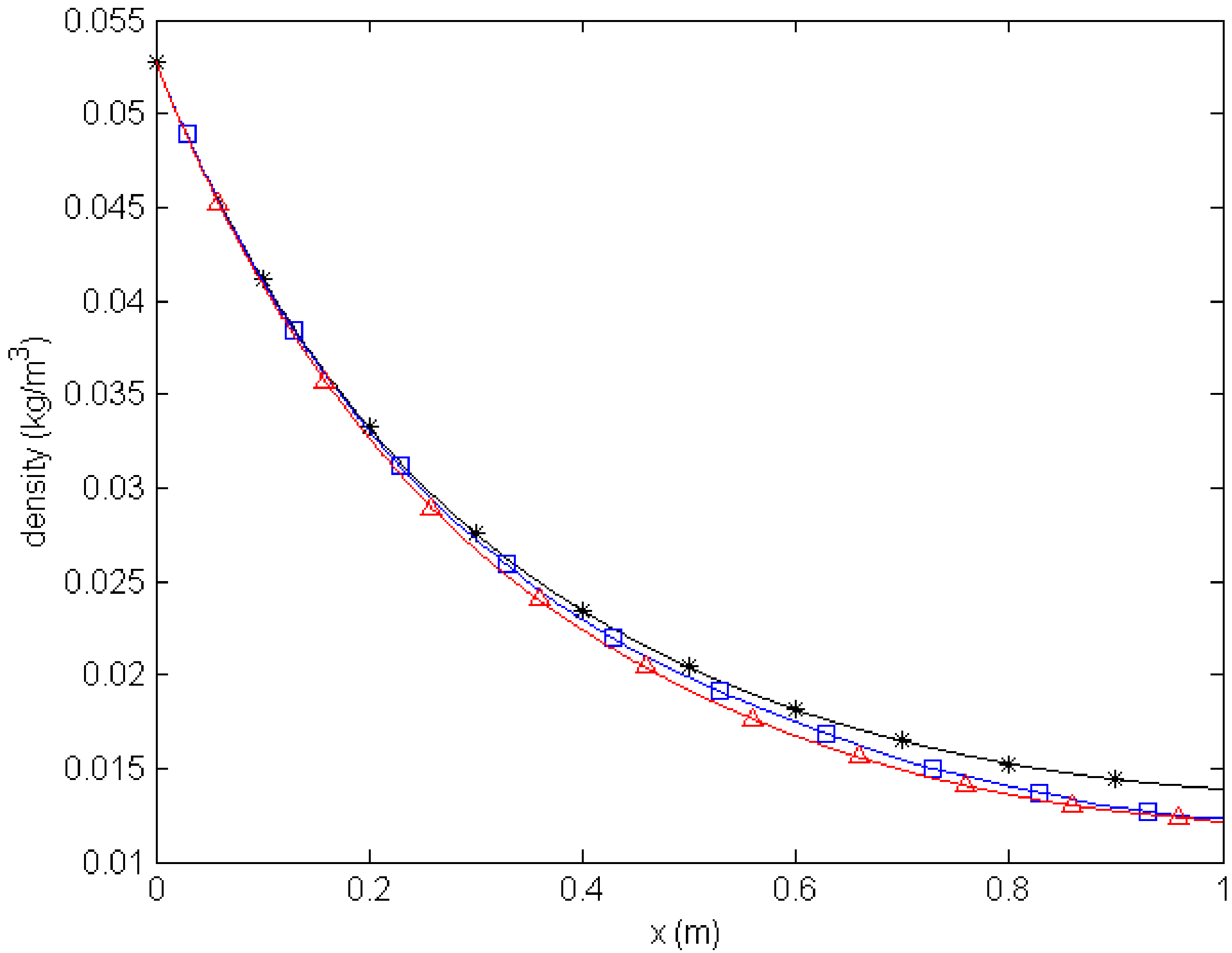
Neural Network Controller

- Choice of a single control two-layer unit or multiple (25) control two-layer units
- Choice of number of hidden layer neurons
- Training algorithm : Resilient Backpropagation
- Inputs : Flow variables (density, velocity and pressure) at the channel inlet.
- Outputs: Electron beam current for the 25 groups of e-beam windows along the channel.

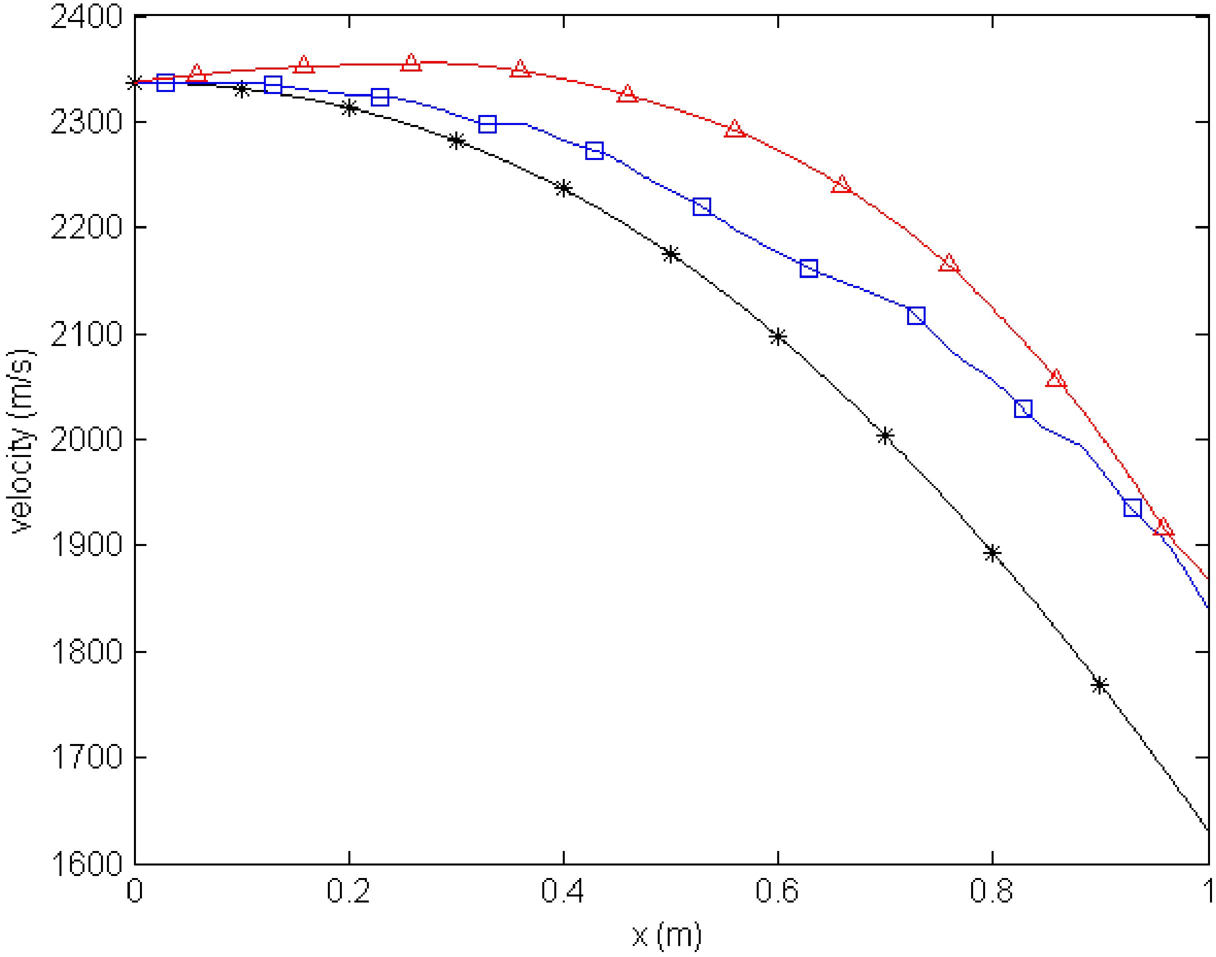
Training the Neural Network Controller

q11 =1e-4, q22 = 1e-4, q33 = 0, p11 = 0, p22 = 0, p33 = 0

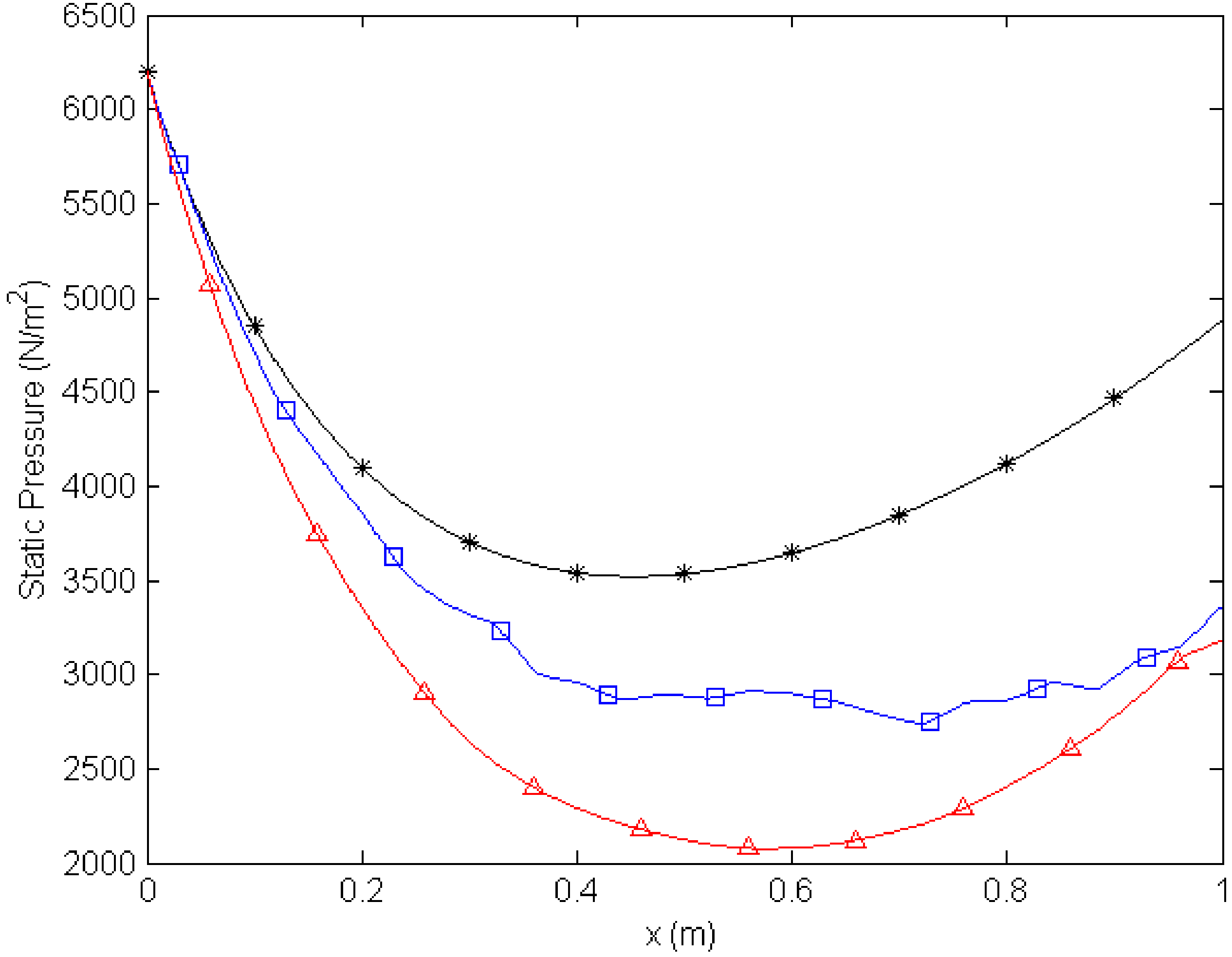




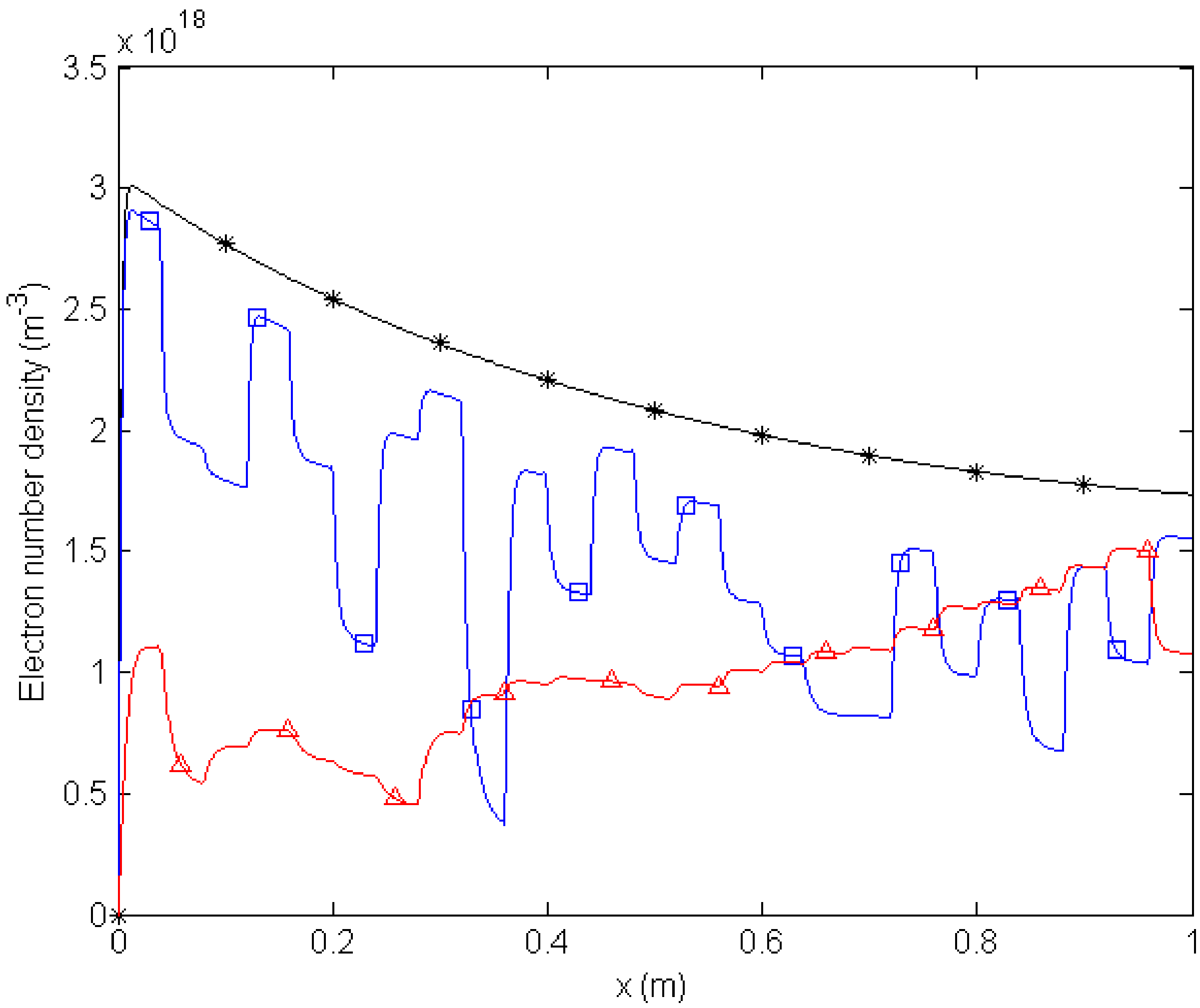
Density profile along the channel, * - Constant current of 50 A/m^2,   - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)



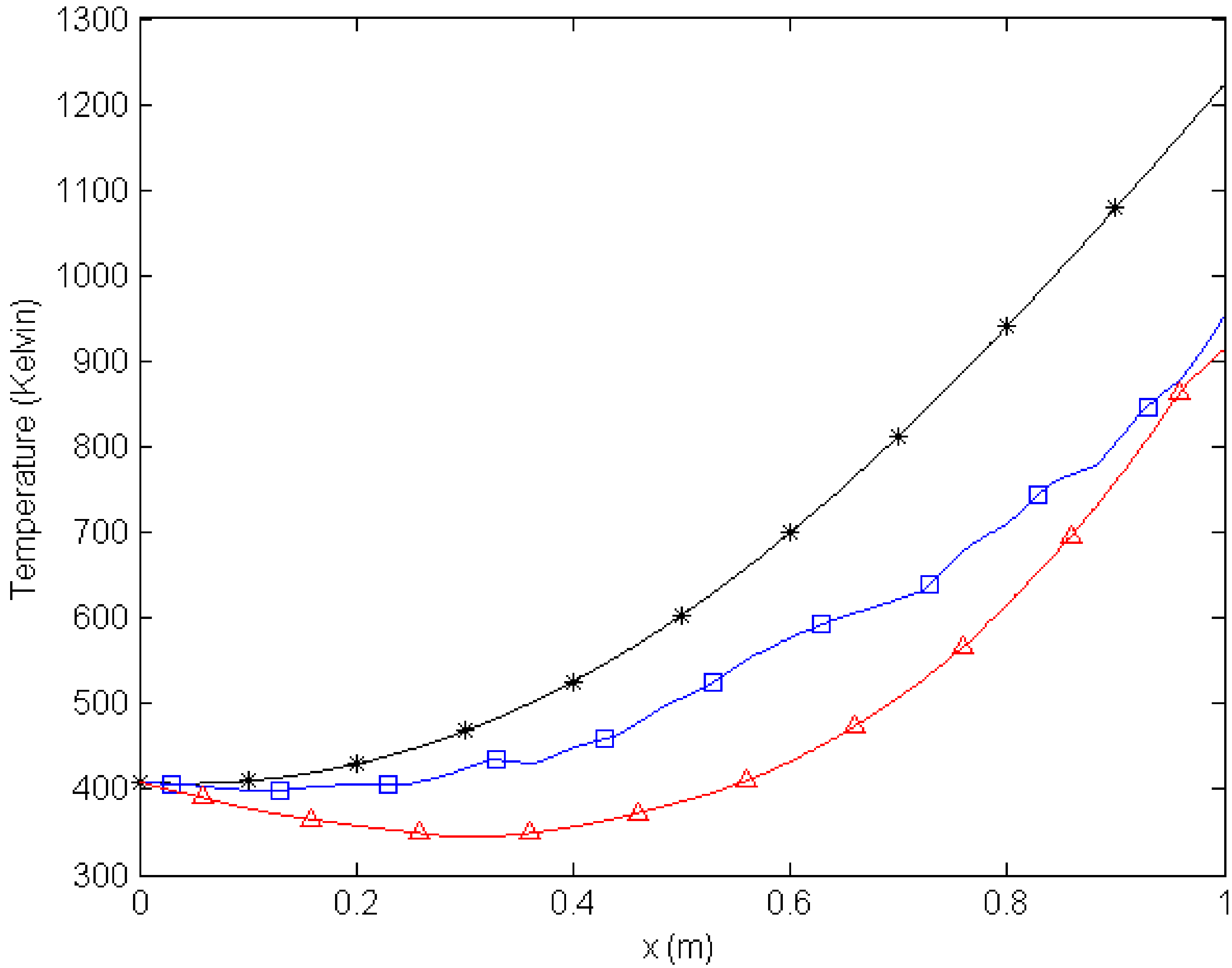
Velocity profile along the channel, * - Constant current of 50 A/m^2, - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)



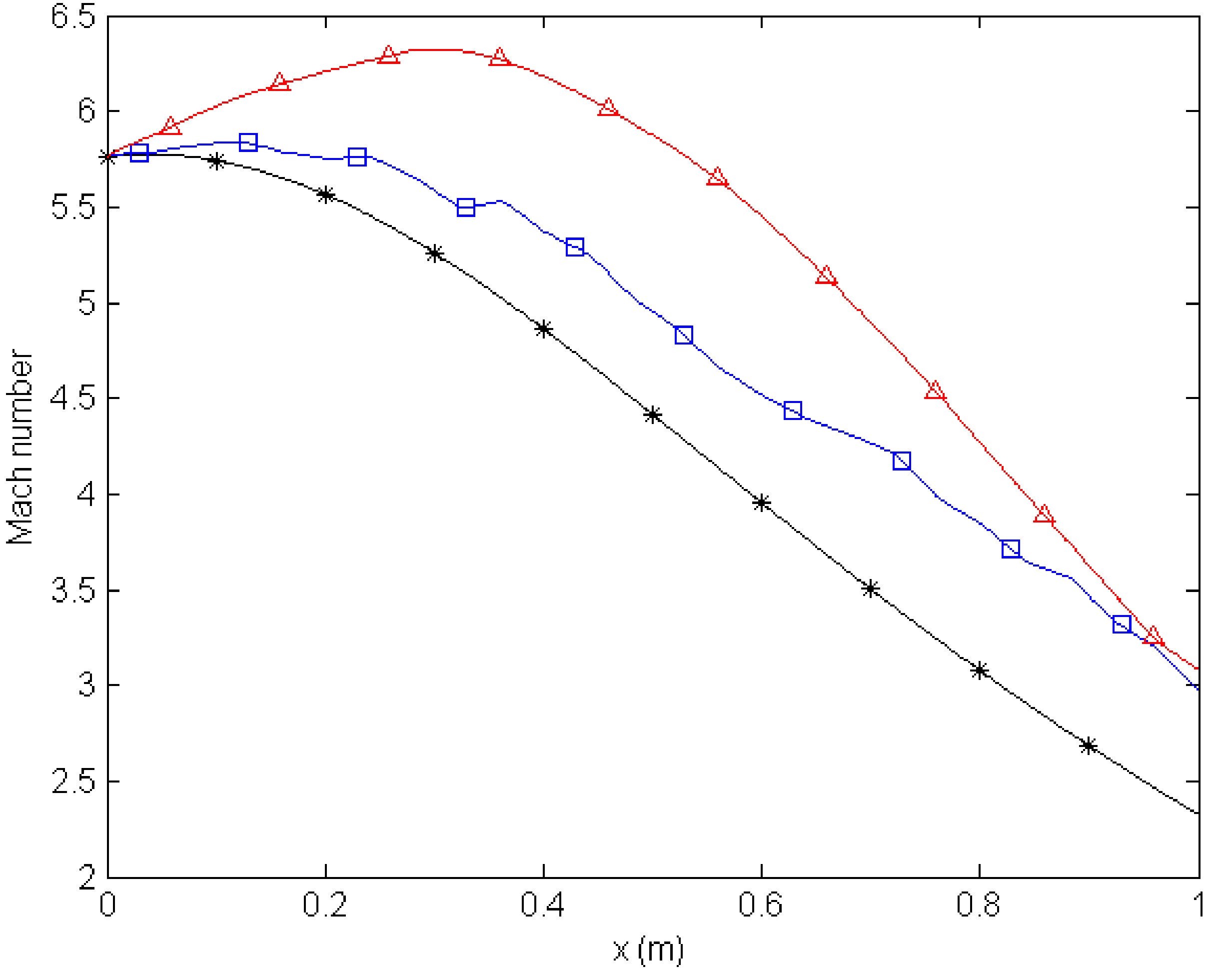
Pressure profile along the channel, * - Constant current of 50 A/m^2,   - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)



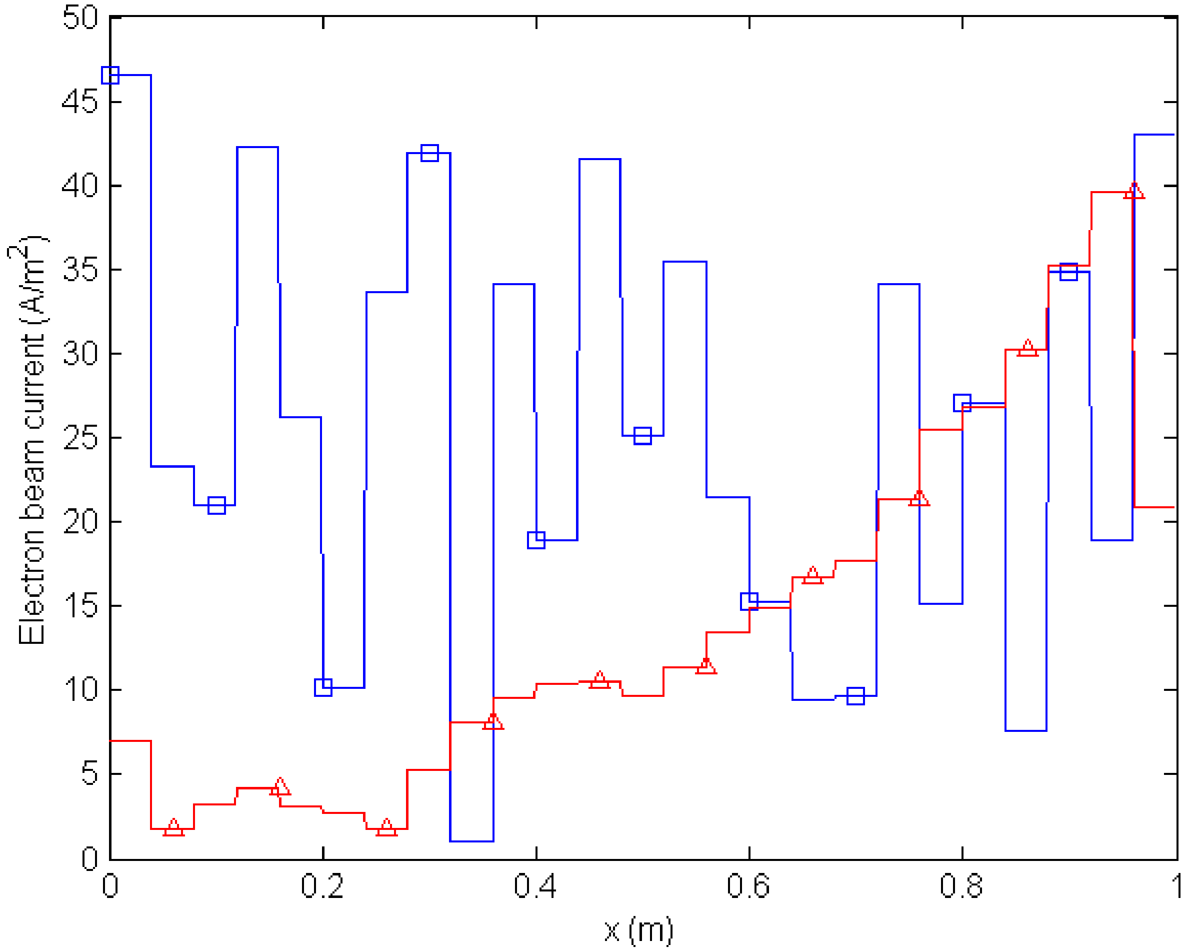
Electron number density profile along the channel, * - Constant current of 50 A/m^2, - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)



Temperature profile along the channel, * - Constant current of 50 A/m^2, - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)



Mach number profile along the channel, * - Constant current of 50 A/m^2, - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)



Electron beam current (control) profile along the channel, * - Constant current of 50 A/m^2, - Random current profile, Δ - Electron beam current profile with the Neural Network controller (q11 = 5*q22)

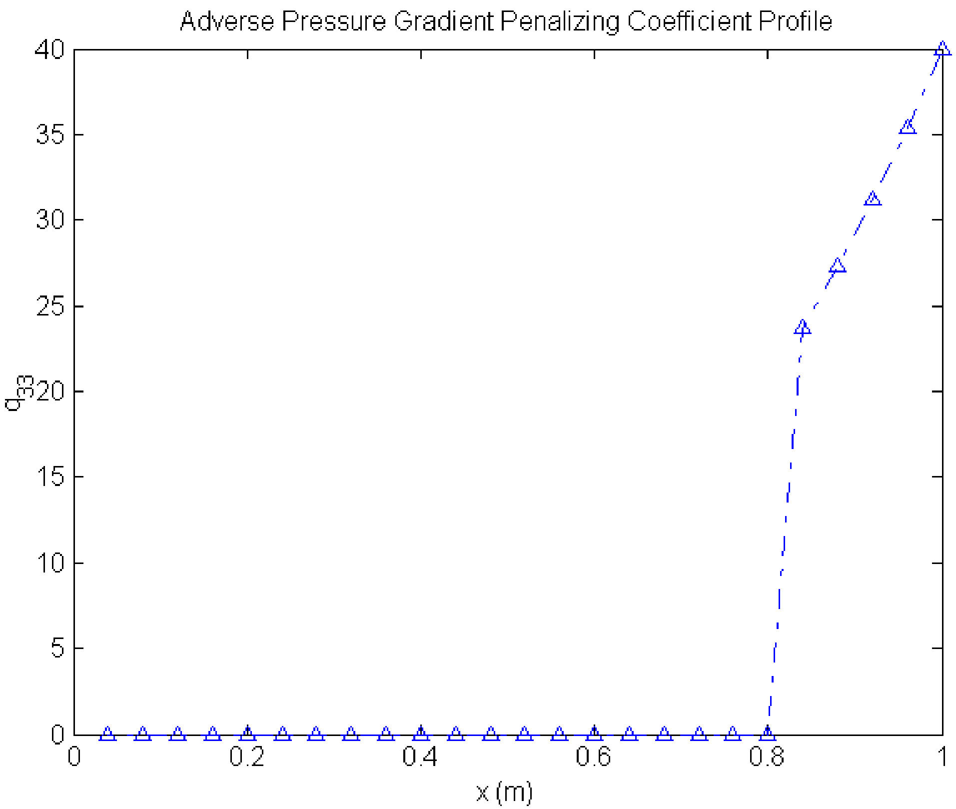
Comparison of different e-beam profiles

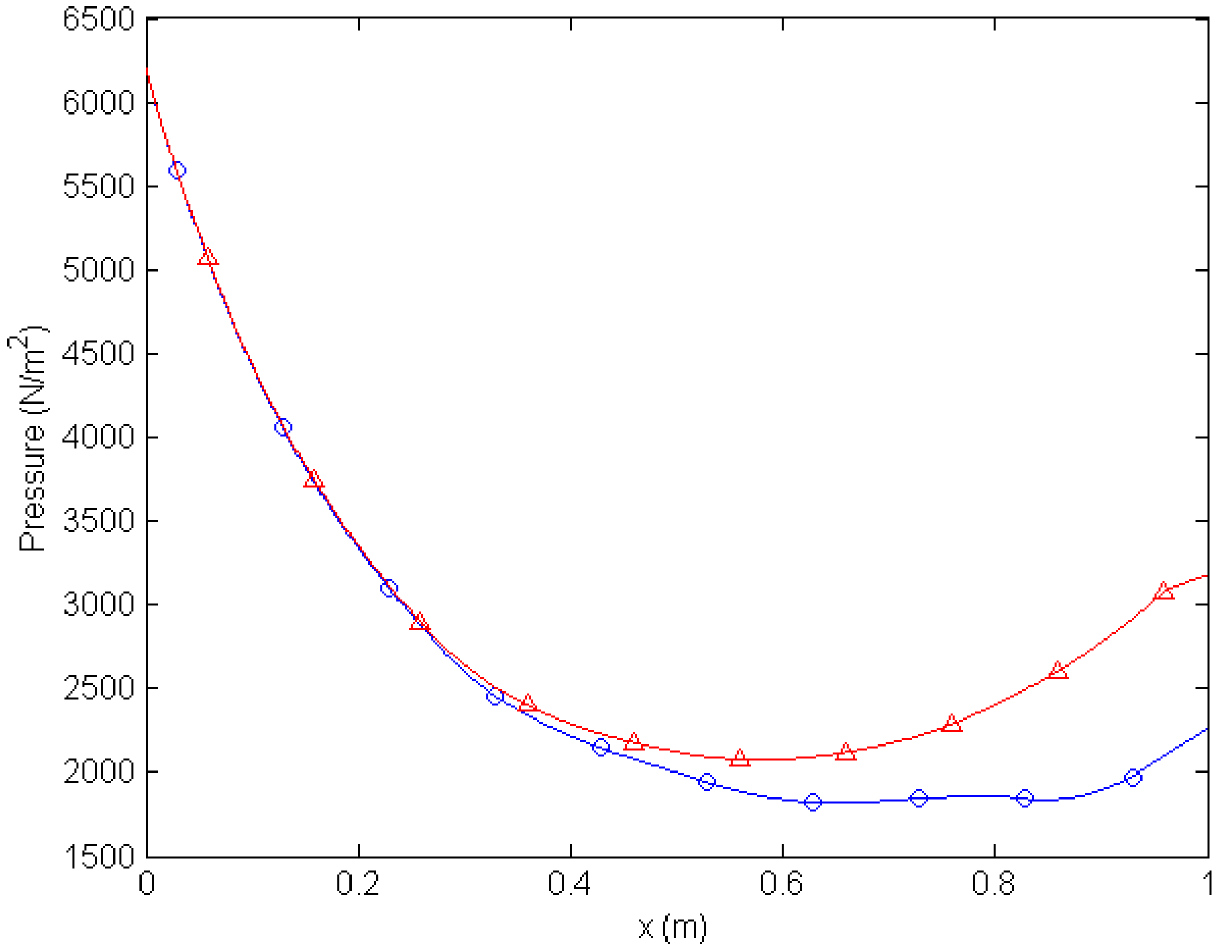
q_{11} (1e-4)	P_{spent} *(max ebc) (kW)*	P_{extr} *(max ebc) (MW)*	$\frac{P_{extr}}{P_{spent}}$	P_{spent} *(random ebc) (kW)*	P_{extr} *(random ebc) (MW)*	$\frac{P_{extr}}{P_{spent}}$	P_{spent} *(opt ebc) (kW)*	P_{extr} *(opt ebc) (MW)*	$\frac{P_{extr}}{P_{spent}}$
1	300.3	1.918	6.39	114.6	1.359	11.86	276.5	1.889	6.83
2	300.3	1.918	6.39	149.8	1.512	10.09	175.2	1.733	9.89
4	300.3	1.918	6.39	148.2	1.569	10.58	109.3	1.513	13.85
6	300.3	1.918	6.39	149.2	1.590	10.65	72.1	1.317	18.27
8	300.3	1.918	6.39	135.5	1.451	10.72	52.2	1.177	22.54
10	300.3	1.918	6.39	145.9	1.492	10.22	39.0	1.055	27.03
12	300.3	1.918	6.39	152.8	1.584	10.36	30.7	0.962	31.33
14	300.3	1.918	6.39	109.6	1.362	12.42	24.4	0.875	35.78
16	300.3	1.918	6.39	150.0	1.544	10.30	19.9	0.801	40.29
18	300.3	1.918	6.39	173.6	1.575	9.07	16.1	0.728	45.28
20	300.3	1.918	6.39	161.2	1.513	9.39	13.1	0.665	50.55

q22 = 1e-4, q33 = 0, p11 = 0, p22 = 0, p33 = 0

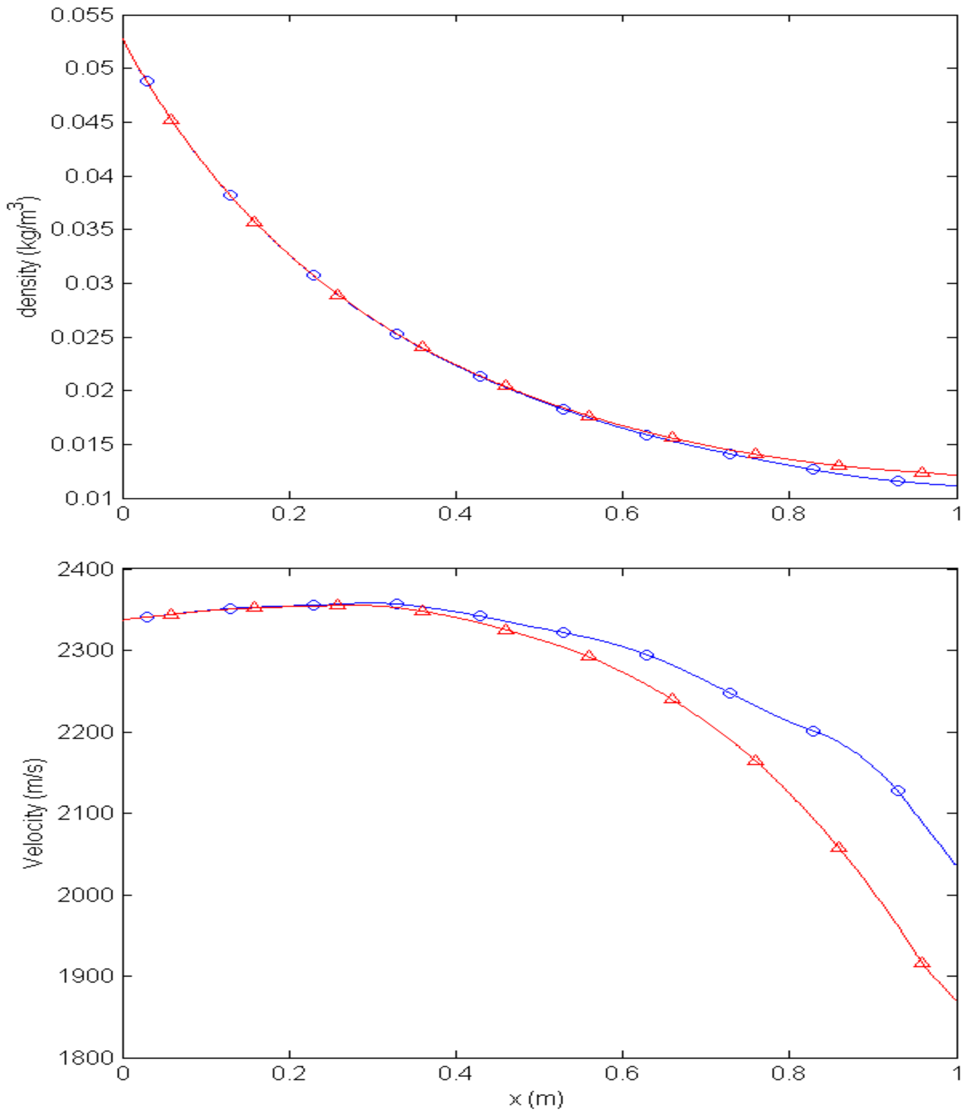
Imposing Pressure Profile Penalty

- q33 relatively weighs the pressure profile
- Adverse pressure gradient towards the end of the channel
- Choose q33 of the form:

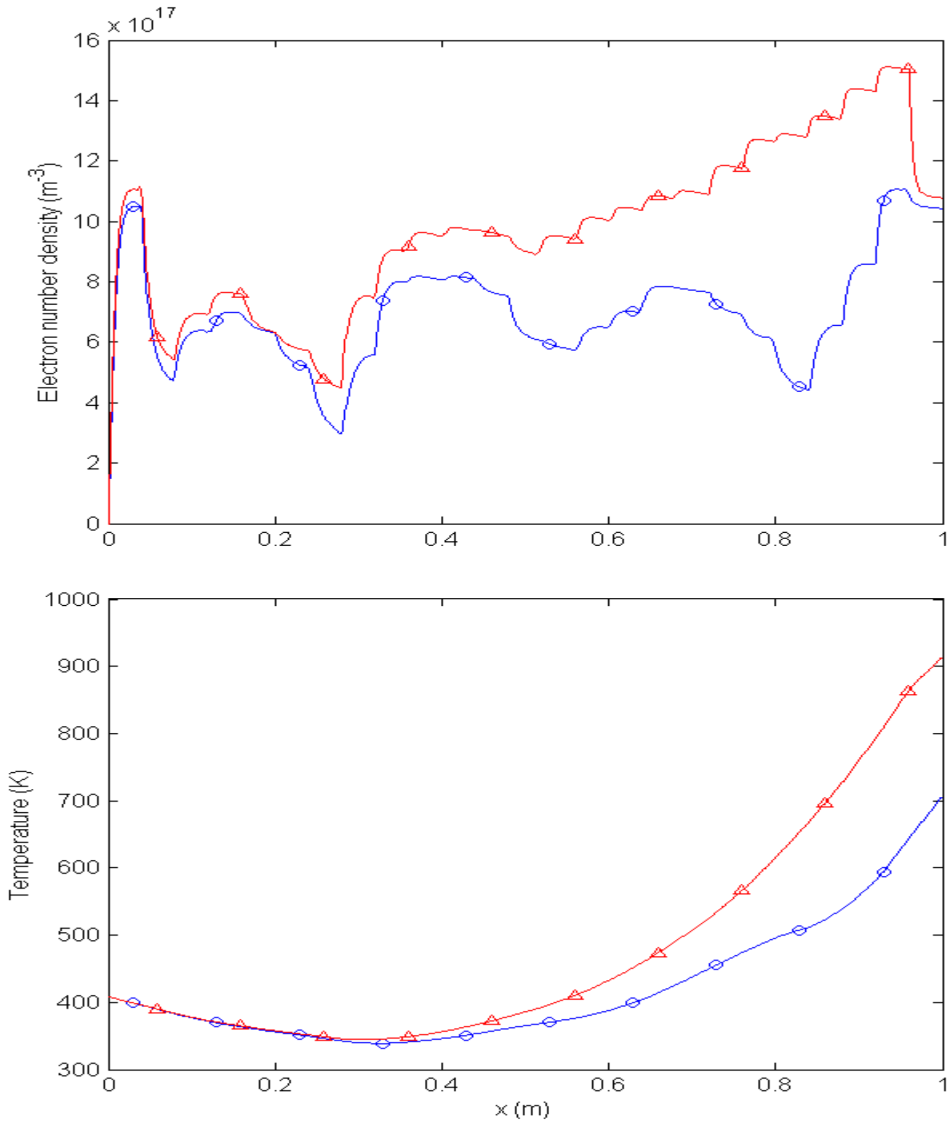




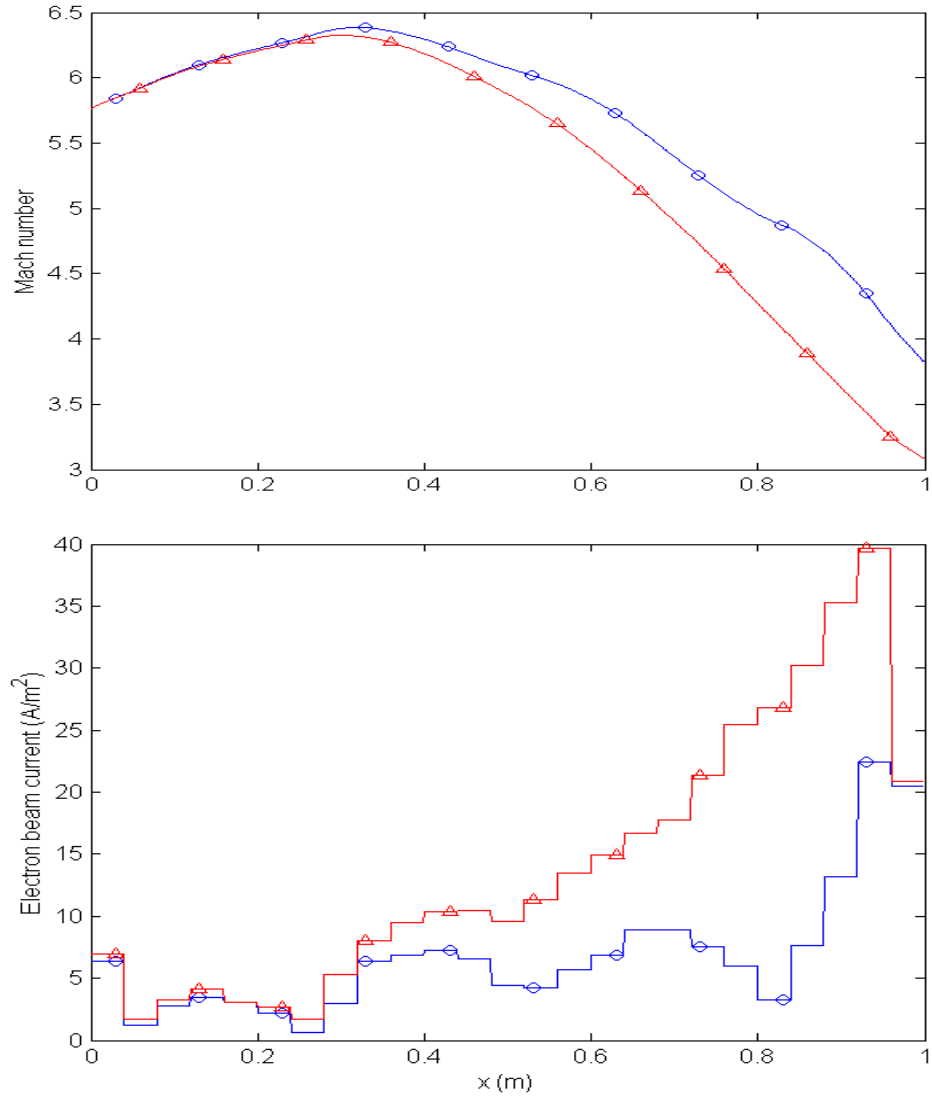
Pressure profile along the channel, Δ - with the NN controller (q11 = 5*q22) without pressure profile penalizing, O – with the NN controller (q11 = 5*q22) with pressure profile penalizing.



Density and velocity profiles along the channel, Δ - with the NN controller without pressure profile penalizing , O - with the NN controller with pressure profile penalizing.



Electron number density and Temperature profiles along the channel, Δ - with the NN controller without pressure profile penalizing, O – with the NN controller with pressure profile penalizing.



Mach number and Electron beam current profiles along the channel, Δ - with the NN controller without pressure profile penalizing, O – with the NN controller with pressure profile penalizing.

Power Analysis

$q11 = 5, q22 = 1e\text{-}4,\ p11 = 0, p22 = 0, p33 = 0$

With q33 = 0

- Power Extracted = 1.411 MW
- Power Spent = 88.27 kW
- Ratio of Power Extracted to Power Spent = 15.99

With q33 penalizing pressure profile

- Power Extracted = 1.043 MW
- Power Spent = 40.97 kW
- Ratio of Power Extracted to Power Spent = 25.47

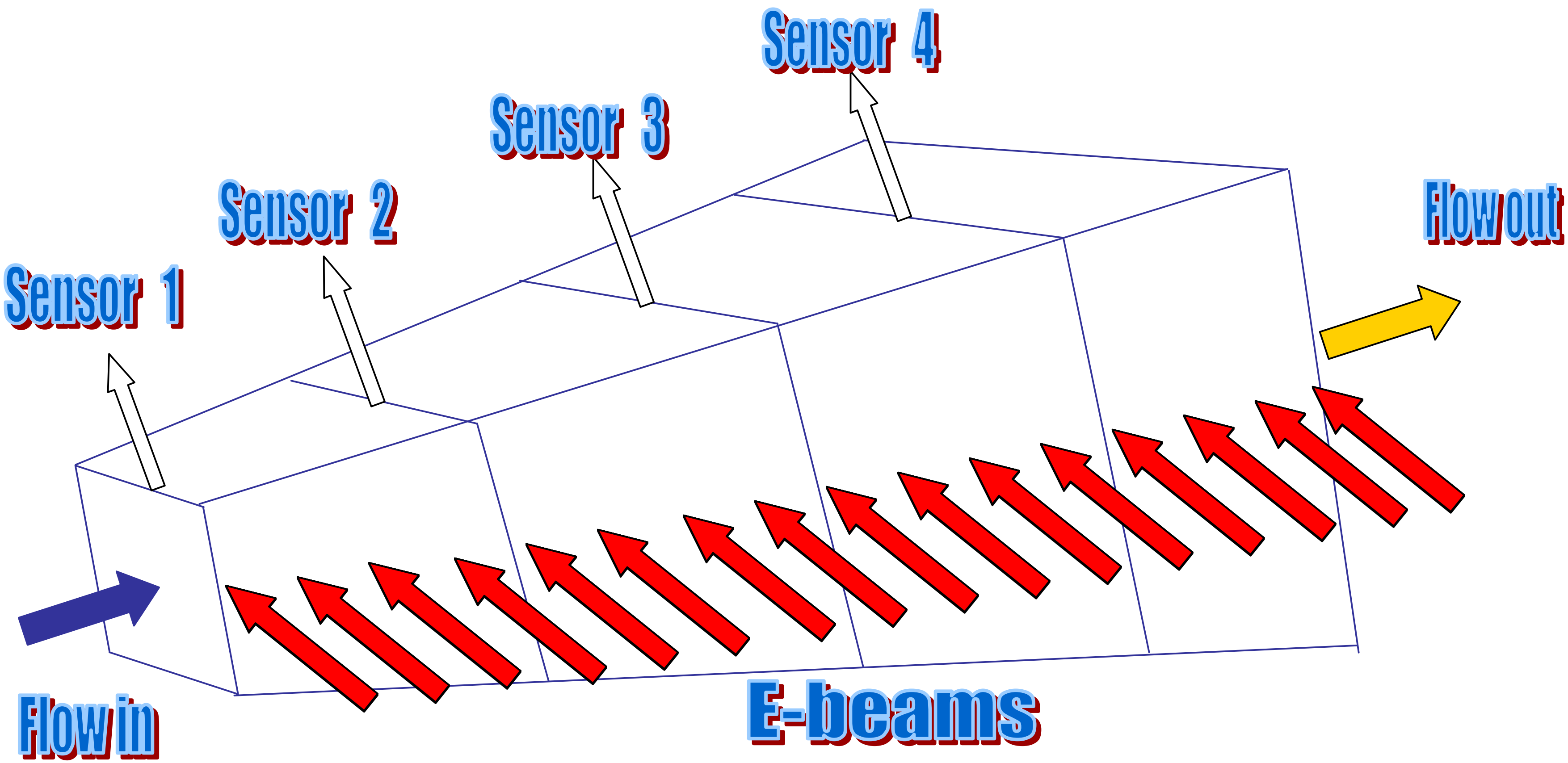
Other Interesting Cases

- Minimizing the Entropy Increase
- Specification of exit Mach number and Temperature
- Electron beam window failures

Future Research Thoughts

- Implementation of a state-feedback form of the controller
  - Using sensor information
  - Computational cost issue for a longer channel
  - Increasing the control resolution

Dynamic Programming Based State-Feedback Architecture



Inlet MHD Channel

Conclusions

- Formulation of the problem of performance optimization of the MHD Generator as an optimal control problem
- Implementation of the open loop architecture in terms of the cost-to-go approach
- Successful training of the CGA and the Controller
- Results for the energy extraction, energy input and the pressure profile terms in the cost function
- Formulation of the dynamic programming based state-feedback form of the control architecture

Acknowledgements

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- Dr. Paul Werbos - National Science Foundation
- Dr. Raymond Chase - ANSER Corporation

Conference Papers:

[1] Kulkarni, N. V. and Phan, M. Q., "Data-Based Cost-To-Go Design for Optimal Control," AIAA Paper No. 2002-4668, AIAA Guidance, Navigation and Control Conference, Monterey, California, August 2002.

[2] Kulkarni, N. V. and Phan, M. Q., "A Neural Network Based Design of Optimal Controllers for Nonlinear Systems," AIAA Paper No. 2002-4664, AIAA Guidance, Navigation and Control Conference, Monterey, California, August 2002.

[3] Kulkarni, N. V. and Phan, M. Q., "Performance Optimization of the Hypersonic Magneto-hydrodynamic Generator at the Engine Inlet," accepted at the AIAA/AAAF 11th International Space Planes and Hypersonics System and Technologies Conference, Orleans, France, September 2002.